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Glacial valley cross-profile morphology, Tian Shan Mountains, China

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Abstract

The morphology of glacial valley cross-sections can be described in terms of power law ($y = ax^b$) or quadratic equations ($y = a + bx + cx^2$) fitted to empirical data. The quadratic solution provides a more robust way of describing the morphology of glacial valley cross-sections, whereas the power law has more potential in understanding the cross-sectional shapes and their evolution. These two functions are used to study the cross-sectional morphology of glacial valleys in the middle and western Tian Shan Mountains and to discuss the comparison with fluvial channels. The major conclusions are: (1) Power law equation parameters (a and b) are sensitive to the origin selection with larger sensitivity in vertical and A ($A = \ln a$) values. Conversely, c values of the quadratic equation are more stable regardless of different origins selected. (2) Hirano and Aniya [Earth Surf. Processes Landforms 13 (1988) 707–716] suggested two characteristic patterns in the relationship between the power law exponent, b, and the valley form ratio, FR. However, glacial valleys in these areas do not fit the Rocky Mountain model for b–FR values described by Hirano and Aniya for alpine glacial valleys. This indicates that this Rocky Mountain model cannot be applied to all alpine glacial areas. (3) The c values of the quadratic equation represent a curvi-linear trend with its corresponding FRs. At the same time, power law parameters (A and b) fit a closed linear relationship both from these areas and others in the published literature. (4) The cross-sectional shapes of glacial valleys show clear differences with fluvial channels by comparing A–b values of glacial valleys with the hydraulic geometry of fluvial channels. This implies that the A–b relationship and the variation range of b values (commonly with 1.5–2.5) may be helpful to differentiate valleys formed by different processes.

Keywords: Glacial valley; Power law model; Quadratic equation; b–FR diagram; Tian Shan Mountains

1. Introduction

The quantitative description of glacial valley cross-profiles is of great significance to morphological research on glacial valleys. It is helpful to capture the essence of glacial valley morphology and to differentiate between valleys formed by different processes. Two principal models have been widely used to describe the morphology of glacial valley cross-sections. One is the power law model ($y = ax^b$), first introduced by Svensson (1959) and widely used in the analysis of glacial valley morphology and its evolution (Graf, 1970; Doornkamp and King, 1971; Jiao, 1981; Liu, 1989; Li et al., 1999). The other is the quadratic equation ($y = a + bx + cx^2$)

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(Wheeler, 1984; Augustinus, 1992; James, 1996). They both have definite advantages and limitations in describing glacial valley cross-sectional shapes, and also their interpretations for morpholometric analyses are very different. Specifically, the quadratic equation is valuable as it provides a steady description of glacial valley cross-sections, but the power law has much more potential in understanding the form of the cross-profile and is more suitable for examining form development or comparing different valley sections (Harbor and Wheeler, 1992; Pattyn and Huele, 1998).

These functions can be used to explore the models of valley development, with the general assumption

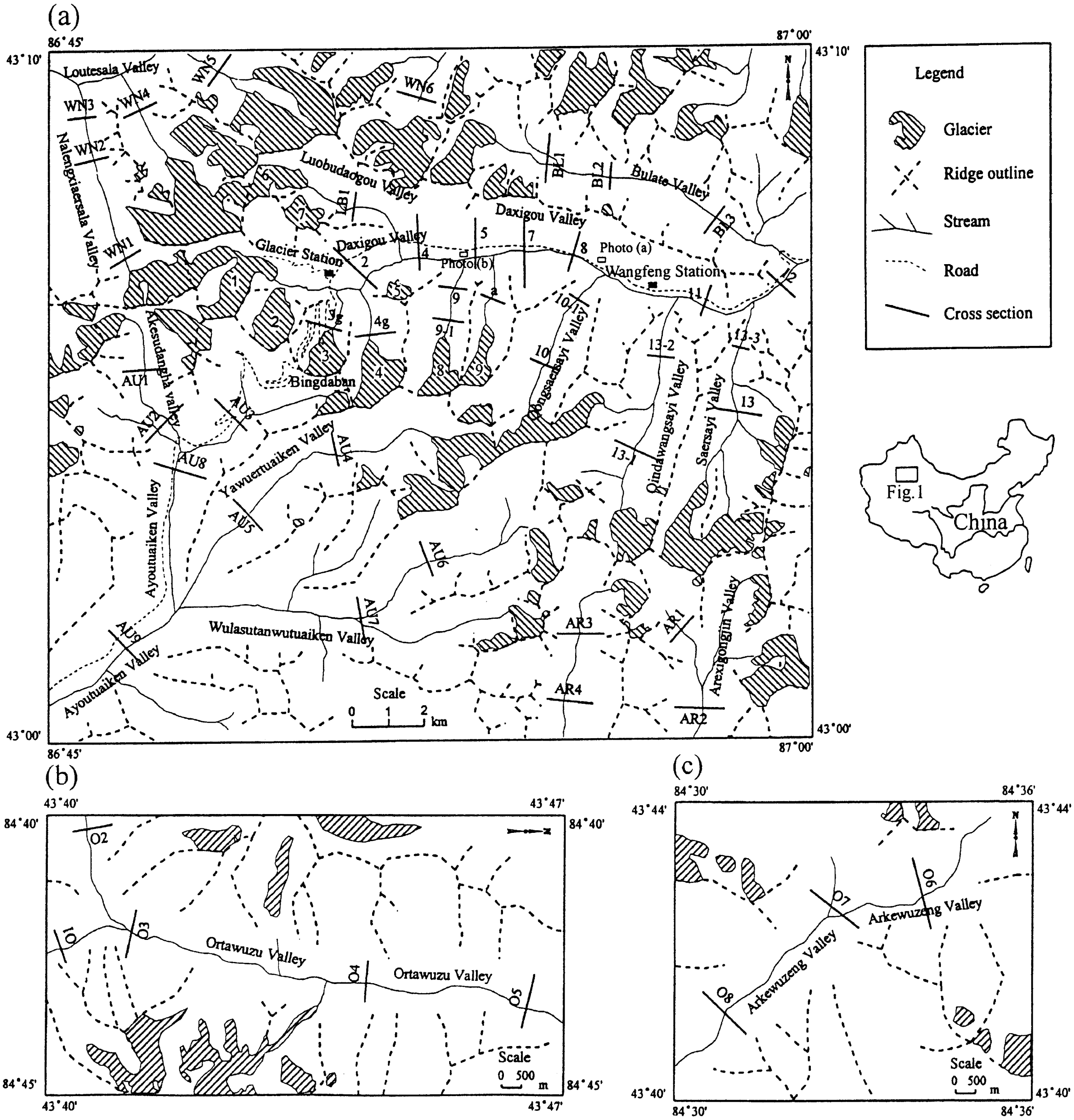

Fig. 1. Locations of cross profiles of glacial valleys in the Tian Shan Mountains, China. (a) Bingdaban area; (b and c) Ortawuzu area.

that form evolves to provide maximum efficiency for ice flow through the section and undergoes a conversion from a V-shape to a U-shape (Flint, 1947; Hirano and Aniya, 1988, 1989, 1990; and discussed by Harbor, 1990). Hirano and Aniya (1988, 1989, 1990) introduced two development patterns on the basis of the relationship between the power law exponent (b) and form ratio (FR) defined by Graf (1970), using cross-profile data from alpine and continental glacial valleys. Harbor (1990) questioned

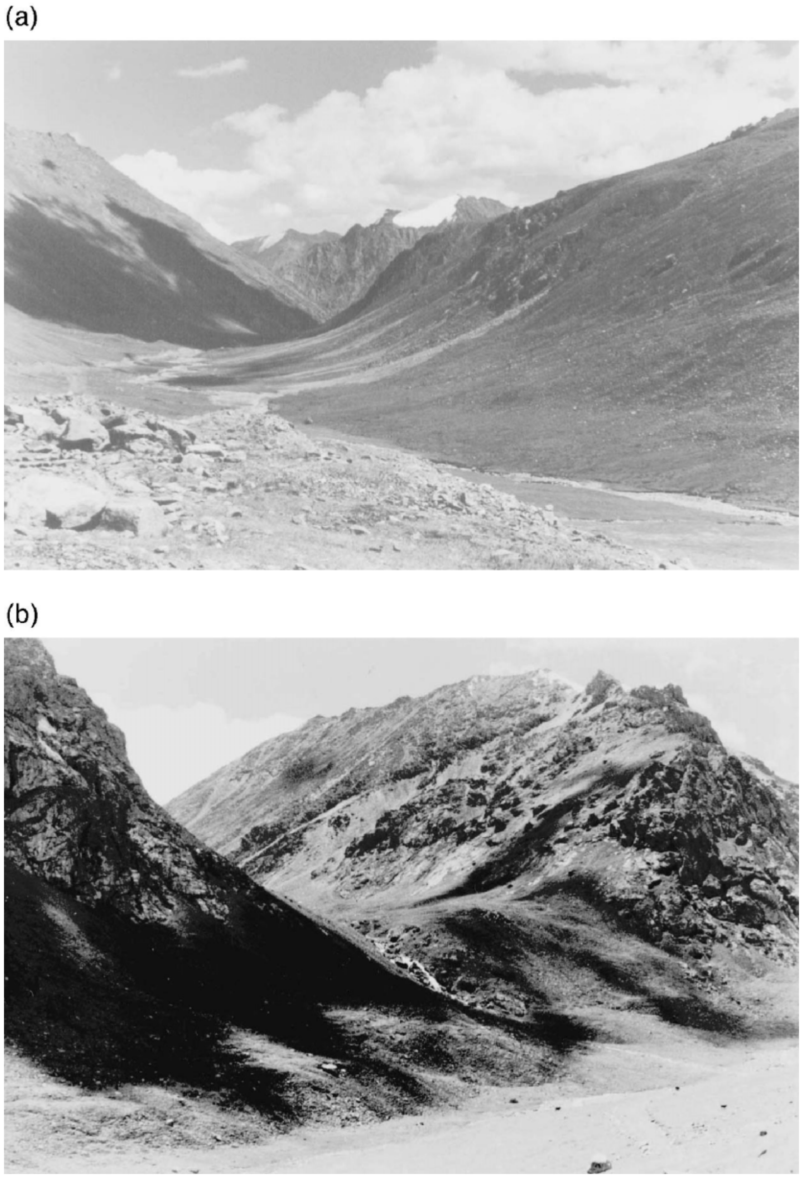

Fig. 2. Some photos of glacial valleys in the Tian Shan Mountains, China: (a) Typical double layer troughs near the Wangfeng Station; (b) Tributary and main valleys near the confluence location between Glacier No.8 Valley and Daxigou Valley. The positions of these photos are indicated in Fig. 1.

some of assumptions used in their research and Augustinus (1992, 1995) pointed out that the influence of bedrock boundary conditions should also be concerned on the development of valley cross-profiles. It is necessary to further examine these patterns in other areas.

The power law equation can also be helpful to differentiate the valleys formed by different processes. Previous work suggested that b values have large variations from less than 1 to over 5 (Graf, 1970; Doornkamp and King, 1971; Wheeler, 1984; James, 1996; Jiao, 1981; Liu, 1989). This range reflects the fact that there are many variations in valley cross-section, depending on geologic complexity or glacial history. It, however, leads to difficulties in describing the cross-sectional morphology of glacial valleys and differentiating valleys formed by different processes because the power law can produce many different forms with variations in a and b values. Does the morphology of glacial valley cross-sections have any characteristic a and b parameters? If the answer is yes, how do we differentiate the valleys formed by glaciers from those formed by other processes? If the answer is no, what are the relationships among these parameters? These are key but not completely resolved issues relating to research on the cross-sectional morphology of glacial valleys.

The purpose of this paper is to use these two equations in studying the morphology of glacial valley cross-sections in the middle and western Tian Shan Mountains, China, to further examine the evolution patterns of glacial valley form, developed by Hirano and Aniya (1988), and discuss their parameter relationships and the comparison with fluvial channels.

2. Study areas

The glacial valleys we studied were selected from the Bingdaban and Ortawuzu areas of the Tian Shan Mountains (Fig. 1). There are double layer troughs in these areas (Cui, 1981a) (Fig. 2). Based on the research on lithostratigraphy, biostratigraphy, geomorphology and radiometric dating at the head of Urumqi River, a north-flowing river in the Bingdaban area (Wang, 1981a), the outer or higher troughs are believed to have formed during the Early Wangfeng Ice Age (probably Oxygen Isotope Stage 8), while the inner troughs, which are cut into the floors of the outer ones, formed in the middle and late Wangfeng Ice Ages (Oxygen Isotope Stages 6 and 2). The inner troughs remain relatively unmodified by subsequent processes, as landforms are modified only slowly in these high altitude areas. Owing to their better preservation, the inner troughs are ideal for studying the morphology of glacial valley cross-sections.

The bedrock in this area is composed mainly of metamorphic rocks such as gneiss and quartz–schist, and secondarily of granitic rocks (Cui, 1981a; Wang, 1981b). Deposits of slope debris and moraine in these glacial valleys are normally 10–20 m thick, but locally, as in the vicinity of Wangfeng Station, they can be up to 40–60 m (Cui, 1981b; Cui et al., 1998). Post-glacial rivers do not cut into the bedrock in most of these glacial valleys. Where rivers do cut into the bedrock, near the Wangfeng Station, the depth of incision is only 2–3 m.

A total of 49 cross-sections of inner troughs including 41 in the Bingdaban area and eight in the Ortawuzu area were sampled from 1:50,000 Chinese Survey topographic maps with 20-m contour intervals (Fig. 1). The trimline of each profile was determined by direct observation of profile convexities and comparison with neighboring cross-sections. Some representative cross-sections of U-shaped valleys that are approximately symmetrical are shown in Fig. 3.

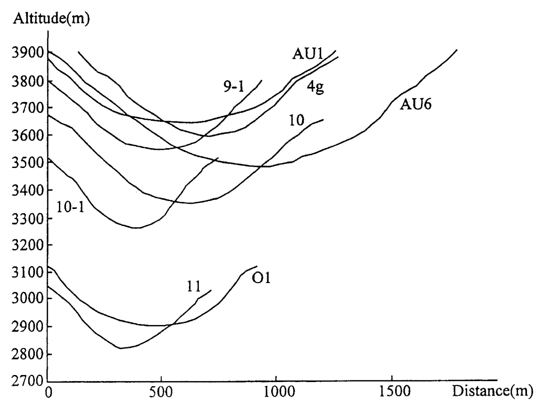

Fig. 3. Some typical cross-sections in the Bingdaban area, Tian Shan Mountains.

3. Method and results

3.1. The power law function

The power law function first used by Svensson (1959) is represented by:

$$y = ax^b \quad (1)$$

where x and y are the horizontal and vertical distances from the lowest point of the cross-section, and a and b are constants. b is commonly used as an index of the steepness of the valley side, and a a measure of the breadth of the valley floor.

This model has been used widely in the analysis of glacial valley morphology and its evolution. It has been found that b values range from less than 1 to over 5, with most values between 1.5 to 2.5 (Graf, 1970; Doornkamp and King, 1971; Aniya and Welch, 1981; Jiao, 1981; Liu, 1989; Li et al., 1999). Furthermore, some studies suggested that the valley morphology progressively approaches a true parabolic form with increasing glacial erosion, and that the stage of valley evolution can thus be gauged by the proximity of b to 2.0 (Svensson, 1959; Graf, 1970; Hirano and Aniya, 1988; Jiao, 1981; Liu, 1989).

The common solution to this function is obtained by transforming it to its logarithmic form ($\ln y = \ln a + b \ln x$). However, this solution leads to several problems including datum problem, bias from logarithmic transformation and post-glacial cut or fill (Wheeler, 1984; Harbor and Wheeler, 1992; James, 1996; Pattyn and Huele, 1998). Quite different results may be obtained by fitting functions to different selections of the origin (x, $y = 0$, 0) or subsets of the valley profile data (James, 1996). So, the determination of the exact shape of the glacial rock surface and selection of the accurate origin are crucial before undertaking this solution. In order to resolve these limitations, Aniya and Welch (1981) provided a solution by repeating the analysis for a large number of points of origin in search for the best possible fit, and Pattyn and Huele (1998) provided another using the general least-squares adjustment associated with taking into account the determination of the coordinates of the origin.

Here, two major steps are used in the analysis of the morphology of glacial valley cross-sections. First, in order to allow for the influence of post-glacial cut or fill, the valley cross-section is smoothed before calculations are undertaken. Specifically: (1) the altitude of the valley bottom is adjusted if the floor is incised by fluvial or other processes; and (2) valley side morphology is extrapolated where talus or till are present especially lateral deposits. The adjustments are rather simple in these areas because deposits on the valley floor and sides are generally less than 10–20 m thick. Even in the vicinity of Wangfeng Station, which has the thickest deposits (40–60 m), in a data set compiled from maps with 20-m contour intervals only one or two points need to be adjusted. Thus, we can easily adjust these points by interpolating using neighboring points. Alternatively, the problem points can be removed without noticeably affecting the calculations. Furthermore, post-glacial rivers do not cut into the bedrock in most parts of these glacial valleys. Where rivers do cut into the bedrock, near the Wangfeng Station, the depth of incision is only 2–3 m, which, also, is not enough to affect the calculations.

Second, Aniya and Welch's (1981) solution is used to resolve the selection of the origin and search for the best possible fit for each profile. This method can also eliminate some influences of deposits on the valley floor. Pattyn and Huele's (1998) solution is not undertaken because it is not suitable for asymmetrical glacial valleys, and even if it could be used to fit each side of the glacial valley cross-section singly, the predicted origin positions from the two sides cannot be guaranteed to be equal either.

Using this method, 41 glacial valley cross-sections in the Bingdaban area and eight cross-sections in the Ortawuzu area (Fig. 1) have been analyzed (Table 1). In order to examine the sensitivity of the parameters to different selections of the origin, the average variations of A ($A = \ln a$) and b values are also calculated, by shifting the origin through specific horizontal and vertical distances (Table 2).

Statistical descriptions of the coefficients of glacial valley cross-sections in these areas and their histograms are shown in Table 4 and Fig. 4. The b values in these areas have a large variation, ranging from 1.027 to 3.503, with most values in the range 1.3–2.5 (71.4%), A values range from -0.414 to

Table 1
Cross-section data (the power law model) for glacial valleys in the Tian Shan Mountains, China

Profile no.	Side 1			Side 2			*FR*	*N*	Profile no.	Side 1			Side 2			*FR*	*N*
	A	*b*	R^2	*A*	*b*	R^2				*A*	*b*	R^2	*A*	*b*	R^2		
O1	−10.985	2.619	0.999	−4.063	1.639	0.998	0.256	10	9	−3.841	1.546	0.999	−12.618	3.046	0.995	0.263	11
O2	−4.389	1.584	0.998	−7.315	2.033	0.999	0.210	10	9−1	−7.305	2.118	0.999	−7.399	2.144	0.998	0.232	9
O3	−4.491	1.614	0.999	−15.189	3.258	0.997	0.212	11	10	−7.159	2.086	0.997	−8.170	2.209	0.996	0.258	14
O4	−4.186	1.637	0.996	−3.532	1.461	0.999	0.269	10	10-1	−7.730	2.291	0.999	−4.407	1.754	0.997	0.304	10
O5	−6.847	1.983	0.993	−0.860	1.118	0.998	0.303	11	11	−2.852	1.484	0.995	−4.348	1.632	0.998	0.316	10
O6	−5.346	1.733	0.998	−10.821	2.619	0.999	0.266	16	12	−3.944	1.629	0.998	−8.245	2.298	0.995	0.278	9
O7	−7.003	1.935	0.991	−0.856	1.05	0.997	0.255	14	13	−9.835	2.359	0.989	−1.369	1.148	0.998	0.216	11
O8	−0.414	1.027	0.998	−11.584	2.69	0.999	0.291	14	13-1	−3.134	1.382	0.998	−9.882	2.468	0.999	0.234	12
AU1	−12.182	2.796	0.989	−9.363	2.307	0.991	0.161	9	13-2	−2.249	1.340	0.998	−7.357	2.129	0.984	0.281	9
AU2	−8.171	2.079	0.999	−8.712	2.198	0.998	0.177	11	13-3	−7.028	2.139	0.998	−4.833	1.805	0.998	0.264	7
AU3	−5.053	1.600	0.999	−2.843	1.355	0.996	0.209	13	BL1	−2.751	1.319	0.999	−10.979	2.533	0.996	0.190	10
AU4	−10.776	2.542	0.997	−5.598	1.720	0.980	0.180	10	BL2	−2.899	1.343	0.997	−1.625	1.193	0.998	0.264	10
AU5	−9.856	2.275	0.986	−2.865	1.338	0.999	0.177	12	BL3	−4.685	1.784	0.999	−5.120	1.830	0.997	0.307	8
AU6	−4.528	1.585	0.998	−10.727	2.470	0.996	0.185	12	LB1	−10.301	2.653	0.999	−4.877	1.662	0.999	0.268	11
AU7	−10.203	2.380	0.997	−15.094	3.023	0.998	0.168	15	AR1	−0.976	1.103	0.999	−7.415	2.054	0.987	0.242	10
AU8	−6.898	1.866	0.994	−11.137	2.571	0.986	0.162	12	AR2	−5.653	1.638	0.980	−4.196	1.547	0.988	0.173	11
AU9	−6.951	1.871	0.997	−8.798	2.053	0.993	0.129	12	AR3	−4.556	1.559	0.988	−3.921	1.534	0.996	0.217	10
3g	−2.945	1.351	0.999	−4.137	1.610	0.995	0.270	12	AR4	−9.155	2.305	0.993	−12.203	2.750	0.982	0.159	10
4g	−9.440	2.368	0.996	−2.680	1.386	0.998	0.246	11	WN1	−5.32	1.784	0.996	−6.324	1.901	0.998	0.232	10
a	−4.649	1.711	0.992	−5.371	1.826	0.991	0.291	10	WN2	−7.196	2.008	0.978	−2.516	1.286	0.996	0.202	9
2	−1.973	1.215	0.997	−12.590	2.568	0.997	0.144	11	WN3	−3.486	1.499	0.981	−2.030	1.249	0.989	0.288	9
4	−1.377	1.193	0.996	−9.247	2.062	0.982	0.155	12	WN4	−2.733	1.297	0.998	−3.063	1.379	0.992	0.230	11
5	−8.768	2.311	0.996	−1.466	1.078	0.996	0.207	10	WN5	−2.905	1.361	0.986	−2.378	1.202	0.976	0.213	10
7	−5.138	1.698	0.996	−6.822	2.106	0.991	0.261	11	WN6	−16.314	3.503	0.989	−1.831	1.159	0.996	0.223	11
8	−2.715	1.372	0.999	−6.574	1.865	0.997	0.233	12									

Table 2
Power law coefficient sensitivity to the selection of the origin for glacial valleys, Tian Shan Mountains, China

Coefficients	Horizonal shifted distance (m)				Vertical shifted distance (m)	
	− 20	− 10	10	20	5	10
A (side 1)	0.936 (16.3%)	0.468 (8.1%)	0.468 (8.1%)	0.936 (16.3%)	0.731 (11.2%)	1.371 (21.0%)
b (side 1)	0.142 (7.3%)	0.070 (3.6%)	0.070 (3.6%)	0.140 (7.2%)	0.116 (5.7%)	0.218 (10.6%)
A (side 2)	0.933 (15.9%)	0.467 (7.9%)	0.466 (7.9%)	0.932 (15.9%)	0.734 (11.2%)	1.377 (21.0%)
b (side 2)	0.139 (7.0%)	0.070 (3.5%)	0.070 (3.5%)	0.142 (7.1%)	0.117 (5.7%)	0.218 (10.6%)

−16.314 and form ratios (FR) vary from 0.129 to 0.316. These results are similar to the previous research in these areas (Jiao, 1981). It was also found that the values of the coefficients of A and b are sensitive to different selections of the origin with larger variation in vertical and A values (Table 2).

3.2. The quadratic equation

The quadratic equation is usually represented by:

$$y = a + bx + cx^2 \quad (2)$$

where y is the elevation above a datum such as sea level, x is lateral distance from the reference station,

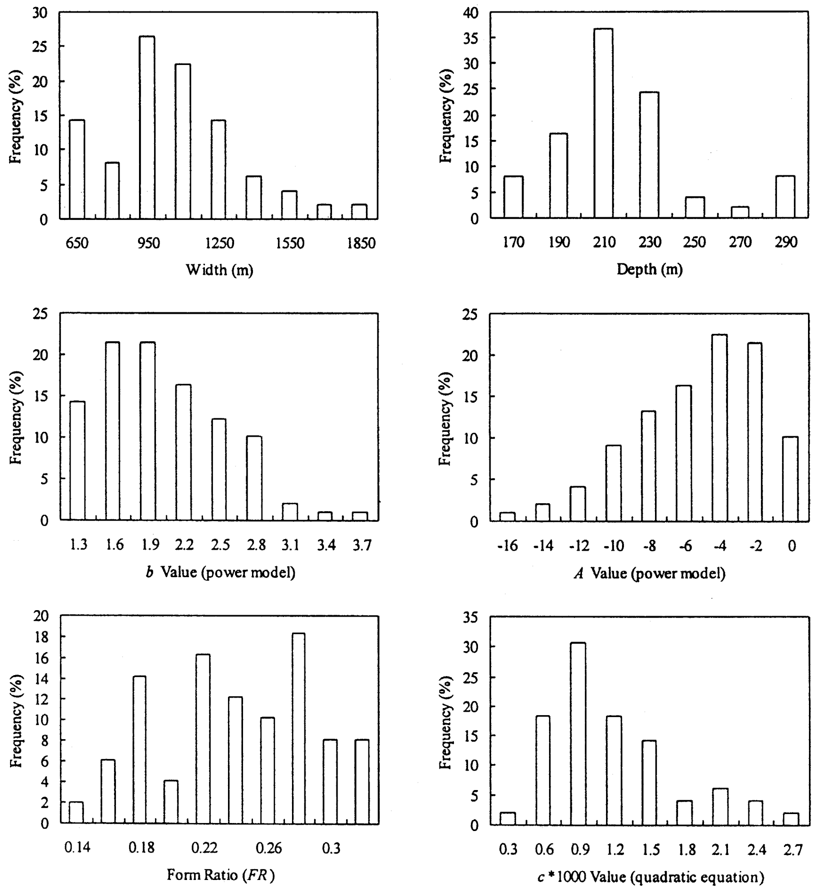

Fig. 4. Histograms of coefficients for glacial valley cross-sections in the middle and western Tian Shan Mountains.

Table 3
Cross-section data (the quadratic equation) for glacial valleys in the Tian Shan Mountains, China

Profile no.	a	b	c	R^2	N	Profile no.	a	b	c	R^2	N
O1	3139.9	−1.12	0.00123	0.991	21	9	3735.8	−1.26	0.00141	0.959	23
O2	3153.4	−0.91	0.00086	0.996	21	9-1	3853.0	−1.29	0.00135	0.997	19
O3	3065.9	−0.92	0.00086	0.980	23	10	3713.6	−1.18	0.00097	0.981	29
O4	2652.7	−0.98	0.00109	0.939	21	10-1	3575.1	−1.68	0.00229	0.982	21
O5	2667.9	−1.45	0.00135	0.984	23	11	3065.4	−1.35	0.00199	0.972	21
O6	3397.3	−1.23	0.00092	0.980	33	12	2975.9	−1.25	0.00184	0.981	19
O7	3104.2	−0.90	0.00074	0.975	29	13	3597.2	−1.03	0.00090	0.993	23
O8	3174.2	−1.45	0.00119	0.993	29	13-1	3746.7	−1.09	0.00087	0.978	25
AU1	3864.4	−0.78	0.00065	0.977	19	13-2	3433.6	−1.46	0.00185	0.987	19
AU2	3855.1	−0.82	0.00051	0.987	23	13-3	3299.3	−1.54	0.00228	0.996	15
AU3	4038.3	−1.05	0.00059	0.983	27	BL1	3779.0	−0.88	0.00071	0.992	21
AU4	3788.4	−0.84	0.00060	0.962	21	BL2	3648.8	−1.27	0.00125	0.963	21
AU5	3713.1	−0.86	0.00056	0.994	25	BL3	3266.5	−1.70	0.00241	0.989	17
AU6	3979.0	−1.10	0.00061	0.994	25	LB1	3860.9	−1.13	0.00113	0.953	23
AU7	3770.6	−0.80	0.00040	0.984	31	AR1	4147.8	−1.50	0.00122	0.987	21
AU8	3725.4	−0.77	0.00049	0.980	25	AR2	3798.2	−0.80	0.00052	0.967	23
AU9	3489.7	−0.61	0.00030	0.991	25	AR3	3983.6	−1.14	0.00088	0.979	21
3g	3930.1	−1.18	0.00113	0.969	25	AR4	3798.5	−0.86	0.00067	0.989	21
4g	3964.7	−1.28	0.00114	0.989	23	WN1	3964.9	−1.24	0.00112	0.994	21
a	3722.8	−1.20	0.00168	0.991	21	WN2	3694.4	−1.03	0.00093	0.977	19
2	3740.5	−0.68	0.00039	0.958	23	WN3	3548.4	−1.20	0.00170	0.985	19
4	3666.6	−0.68	0.00041	0.871	25	WN4	3660.1	−1.02	0.00088	0.981	23
5	3466.2	−0.75	0.00077	0.923	21	WN5	3761.9	−0.89	0.00082	0.969	21
7	3530.4	−1.26	0.00127	0.960	23	WN6	3641.5	−0.83	0.00081	0.904	23
8	3464.3	−1.00	0.00087	0.990	25						

and a, b, and c are coefficients (Wheeler, 1984; Augustinus, 1992; James, 1996). In this model, the coefficients a and b control the position of the valley cross-section in the coordinate system and have not direct linkage with the valley form, the valley form is mainly controlled by the c value, and the larger c is, the narrower valley floor is. The quadratic equation provides a succinct and explicit description of the entire valley form, but this approach has the a priori assumption that the cross-profile form is parabolic and symmetrical (Harbor and Wheeler, 1992; James, 1996). It may lose some ability in describing asymmetrical valleys and contributes less to the understanding of valley form evolution.

We also use the quadratic equation to analyze the cross-sectional shapes of glacial valleys in these areas (Table 3). The c values in these areas range from 0.00030 to 0.00241 (Table 4), with most values in the range 0.00060–0.00120 (49%) (Fig. 4). Com-

Table 4
Statistical descriptions of coefficients of glacial valleys, Tian Shan Mountains, China

	The power model		The quadratic equation	Width (m)	Height (m)	FR
	A	b	c			
Average	−6.170	1.869	0.00105	977.0	210.6	0.230
RMS[a] error	3.597	0.539	0.00052	280.4	30.6	0.048
Maximum	−16.314	3.503	0.00241	1721.3	290	0.316
Minimum	−0.414	1.027	0.00030	518.8	160	0.129

[a]RMS = root mean square

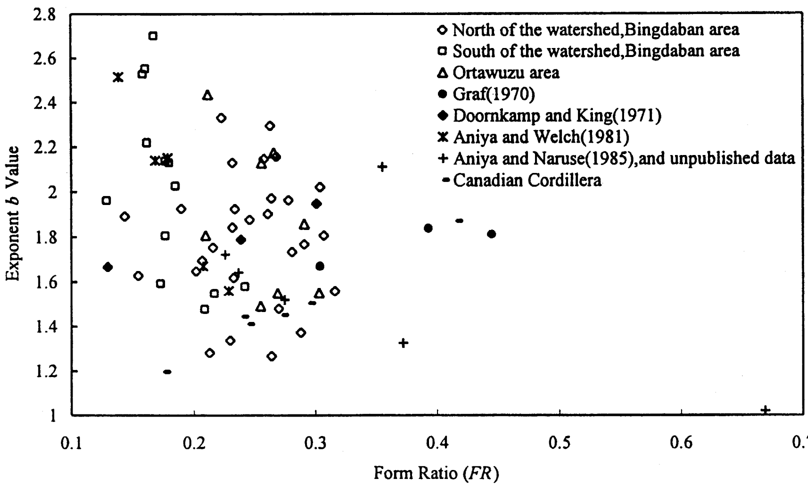

Fig. 5. The *b*–*FR* diagram for glacial valley cross-sections in the middle and western Tian Shan Mountains.

pared with the power law function, this solution provides a more robust way in describing the morphology of glacial valley cross-sections, especially in relation to the influence of different selections of the origin. If we omit the origin point in the calculation, the average variation of c values is very small (0.000017), only 1.68% in relative errors.

4. Discussion

4.1. The b–FR diagram of the power law model

Hirano and Aniya (1988) studied the relationship between b and FR (b–FR diagram), and arrived at two models to fit the data available in the literature and from their own work. One is the Rocky Mountain model, which describes the development of alpine valleys with b values becoming larger with increasing FRs (deepening without widening), and the other is the Patagonia–Antarctica model describing the development of glacial valleys formed by a continental ice with larger b values associated with smaller FRs (widening without deepening).

Although Harbor (1990) questions whether the ergodic approach used to develop these two models is appropriate, Hirano and Aniya's models provide an important observation that there are two distinct relationships between b values and form ratios. Augustinus (1992) examined this relation using data from the New Zealand Southern Alps and pointed out that the influence of bedrock boundary conditions should also be concerned with the development of valley cross-profiles.

The b–FR diagram of glacial valleys in the Tian Shan Mountains (Fig. 5) (the b value used here is the average value for the two valley sides) does not fit the Rocky Mountain model of Hirano and Aniya

Table 5
The average b and FR values in the Daxigou Valley and Ayoutuaiken Valley, Tian Shan Mountains, China

	Daxigou Valley			Ayoutuaiken Valley	
	Tributaries	Upper main	Down main	Tributaries	Main
Average b	1.918 ± 0.230	1.855 ± 0.208	1.713 ± 0.218	2.129 ± 0.312	2.255 ± 0.316
Average FR	0.260 ± 0.027	0.207 ± 0.057	0.276 ± 0.042	0.176 ± 0.010	0.159 ± 0.021

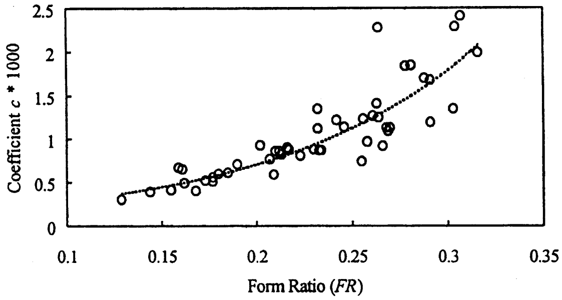

Fig. 6. The relationship between c and FR of glacial valley cross-sections in the middle and western Tian Shan Mountains.

(1988) and has a slight trend of becoming smaller with increasing FRs similar to the Patagonia–Antarctica model. Specifically, the average b value of tributaries is smaller and FR value is larger than those of main valleys in the Ayoutuaiken valley in the southwest of the Bingdaban area. In the other case, the Daxigou valley in the north of the Bingdaban area, however, the b–FR relationship is rather complex. The average b and FR values for tributaries are larger than for upper main valleys, and the average b decreases and FR increases from upper to down main valleys (Table 5). The same characteristic in these two valleys is that FRs decrease definitely from tributaries to main valleys seemly showing a more efficient widening process than overdeepening in these areas. This indicates that the b–FR relationship of alpine glacial valleys is complex and does not just conform to one specific Rocky Mountain model.

4.2. The c–FR relationship of the quadratic equation

As noted before, in the quadratic equation, only c values reflect the shape of the valley cross-section. As the b–FR diagram of the power law, the relationship between c and FR is also important in revealing the morphology of glacial valley cross-sections. The c and FR values in these areas (Fig. 6) result in a curvi-linear trend with larger c values associated with larger form ratios. Since the average FR values of tributaries are commonly larger than main valleys in these areas, the valley floors of tributaries are generally narrower (larger c values) than main valleys.

4.3. The A–b relationship of the power law model

The morphology of a glacial valley cross-section is controlled by the from ratio and the two parameters of the power function (a and b). As noted earlier, b values have large variation ranges from less than 1 to over 5 from the published data (Graf, 1970; Doornkamp and King, 1971; Wheeler, 1984; James, 1996; Jiao, 1981; Liu, 1989) and this paper. Although this range reflects the fact that there are many variations in valley cross-section, depending on geologic complexity or glacial history, it results

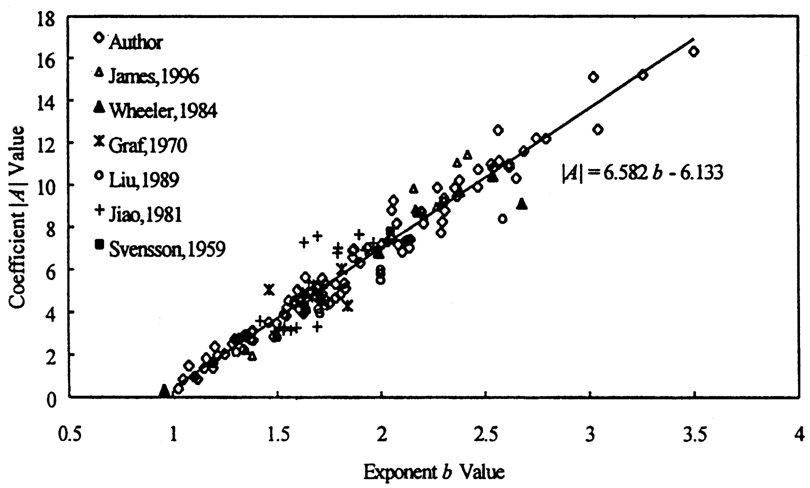

Fig. 7. The A–b relationship of glacial valley cross-sections using data both from this paper and other published sources.

in difficulties in describing the glacial valley cross-section morphology and differentiating between valleys formed by different processes, because the power law can produce many different forms with variations in a and b values. The previous work commonly uses the b value and ignores the a value in describing and comparing different valley cross-sections, but the valley form could be very different associated with different a values even with the same b values (Graf, 1970). A complete description of valley cross-section needs to seek both a and b values and the relationship between a and b is also important. However, the relationship between a and b has not received due attention, except for the qualitative result that as the order of the valley segments increases from first to second to third, both b and a increase (Graf, 1970).

Because a is generally far less than 1, we use its logarithm value (A) to explore the relationship between a and b. When we plot the $|A|$ values against their corresponding b values, a clear linear trend is found, with b values becoming larger associated with increasing $|A|$s (Fig. 7). The regression function in the Tian Shan Mountains is:

$$|A| = 6.582\,b - 6.133 \tag{3}$$

It is a strong regression function with the explained variance (R^2) being 0.97. The data shown in Fig. 7 comes from different alpine glaciated areas

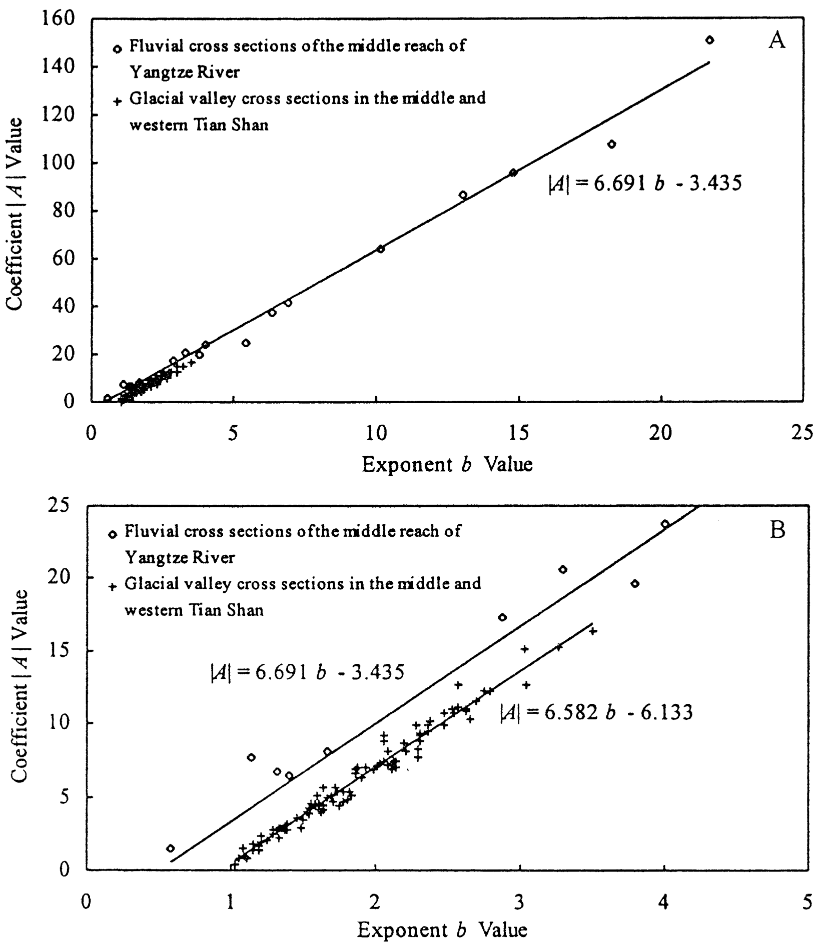

Fig. 8. The A–b relationship of cross-sections of glacial valleys and fluvial channels.

both in China and other countries, and from different published sources.

This relationship reflects a mathematical certainty of the power law function with probably the same scale in width and depth. This implies that most of glacial valleys are of the same scale in width and depth. It also provides a basis for any one parameter to be used to describe the valley form. The other can be ignored because A and b are strongly related. Furthermore, the A–b linear relationship and the variation range of b values (commonly with 1.5–2.5) may be helpful to differentiate valleys formed by different processes.

4.4. Comparison with fluvial channels

One reason for the use of quantitative equations to describe glacial valley cross-sections is that the quantitative method can be helpful to differentiate the valleys formed by different processes. In this paper, the comparison of glacial valleys and fluvial channels is discussed by using the power law function.

Graf (1970) derived the power function of the glacial valley cross-section as an analogy with fluvial hydraulic geometry. Correspondingly, power law parameters (a and b) of fluvial channels can also be calculated.

From fluvial hydraulic geometry we derive:

$$W = \alpha_1 Q^{\beta_1}$$

$$h = \alpha_2 Q^{\beta_2} \tag{4}$$

where W, h, and Q are the channel width, depth and discharge, respectively, and α_1, β_1, α_2, β_2 are constants. After transformation, we acquire:

$$h = \left[\frac{\alpha_2}{\alpha_1^{\left(\frac{\beta_2}{\beta_1}\right)}} \right] W^{\left(\frac{\beta_2}{\beta_1}\right)} \tag{5}$$

If the channel is symmetric, substituting W with $2x$, we get:

$$h = \alpha_2 \left[\frac{2}{\alpha_1} \right]^{\left(\frac{\beta_2}{\beta_1}\right)} x^{\left(\frac{\beta_2}{\beta_1}\right)} \tag{6}$$

Eq. (6) is a power law form and can be represented as $y = ax^b$, and the solutions to a and b are:

$$a = \alpha_2 \left[\frac{2}{\alpha_1} \right]^{\left(\frac{\beta_2}{\beta_1}\right)} \tag{7}$$

$$b = \frac{\beta_2}{\beta_1} \tag{8}$$

Here, we calculate 17A ($A = \ln a$) and b values of fluvial channel cross-sections on the basis of hydraulic geometry research in the middle and lower reach of Yangtze River (You et al., 1985). It is shown that A and b values for fluvial channel cross-sections also conform to a linear relationship ($|A| = 6.691b - 3.435$, $R^2 = 0.99$) like the glacial valleys (Fig. 8), but there are evident differences between fluvial channels and glacial valley cross-sections.

(1) The range of A and b values for fluvial channel cross-sections is evidently larger than that for glacial valleys (channels). The b values in the range 1.5–2.5 are most common for glacial valley cross-sections, but they range from far less than 1 to several hundreds in fluvial channels. This indicates that the morphology of fluvial channel cross-sections has larger variations than that of glacial valleys.

(2) The A–b relationship for the two valley channel types is clearly different. As shown in Fig. 8, the slope of A–b relationship of the two valley channel types is about the same, but the intercept for fluvial channels is larger than that for glacial valleys. The $|A|$ values for fluvial channels are larger than those for glacial valleys with the same b values. This indicates that the bottom in fluvial channels is relatively wider than that of glacial valleys.

The fluvial channel cross-sections discussed here are only from lowland river channels, and the differences between glacial valleys and other type of valleys need further research.

5. Conclusion

The morphological characteristics of glacial valley cross-sections in the middle and western Tian Shan Mountains, China, are discussed here using both power law and quadratic equations. The major conclusions are as follows.

(1) Power law coefficients (A and b) have large variations. b values in these areas range from 1.027 to 3.503, with most values in range 1.3–2.5 (71.4%), and A values range from -0.414 to -16.314. At the same time, they are sensitive to different selections for the origin with larger variation in vertical and A values. Conversely, the quadratic solution provides a more robust way in describing the morphology of glacial valley cross-sections. Its c values range from 0.00030 to 0.00241 and with rather small variation (0.000017) associated with the selection of the origin.

(2) The b–FR diagram in these areas does not confirm to the Rocky Mountain model of Hirano and Aniya (1988) but with similar trend of the Patagonia–Antarctica model. The average FR values of tributaries are commonly larger than those for the main valleys in these areas. In the quadratic equation, its coefficient c and the form ratio show a curvilinear trend with larger c values associated with larger FRs.

(3) The relationship between A ($A = \ln a$) and b of the power law for the glacial valley cross-sections represents a close linear relationship, fitted from data from both this paper and other published sources. This relationship and the variation range of b values (commonly with 1.5–2.5) may be helpful to differentiate valleys formed by different processes. The A–b relationship of fluvial channel cross-sections in the middle and lower reaches of Yangtze River shows trends similar to glacial valleys, but the variation of b values for fluvial channel cross-sections is much larger than for glacial valleys. At the same time, the relative bottom width of fluvial channel cross-sections is larger than that of glacial valleys.

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