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An Improved Method Based on Shallow Ice Approximation to Calculate Ice Thickness along Flow-Line and Volume of Mountain Glaciers

Huilin Li* (李慧林)
State Key Laboratory of Cryospheric Sciences/Tianshan Glaciological Station, Cold and Arid Regions Environmental and Engineering Research Institute, Chinese Academy of Sciences, Lanzhou 730000, *China*
Zhongqin Li (李忠勤)
State Key Laboratory of Cryospheric Sciences/Tien Shan Glaciological Station, Cold and Arid Regions Environmental and Engineering Research Institute, Chinese Academy of Sciences, Lanzhou 730000, *China*;
College of Geography and Environment Sciences, Northwest Normal University, Lanzhou 730070, *China*
Mingjun Zhang (张明军), **Wenfeng Li** (李汶峰)
College of Geography and Environment Sciences, Northwest Normal University, Lanzhou 730070, *China*

ABSTRACT: To evaluate the water storage and project the future evolution of glaciers, the ice-thickness of glaciers is an essential input. However, direct measurements of ice thickness are laborious, not feasible everywhere, and necessarily restricted to a small number of glaciers. In this article, we develop a simple method to estimate the ice-thickness along flow-line of mountain glaciers. Different from the traditional method based on shallow ice approximation (SIA), which gives a relationship between ice thickness, surface slope, and yield stress of glaciers, the improved method considers and presents a simple way to calibrate the influence of valley wall on ice discharge. The required inputs are the glacier surface topography and outlines. This shows the potential of the method for estimating the ice-thickness distribution and volume of glaciers without using of direct thickness measurements.
KEY WORDS: improved method, shallow ice approximation, ice thickness calculation, glacier volume calculation, mountain glacier.

INTRODUCTION

There are 46 377 glaciers with a total area of 59 425 km^2 and 5 600 km^3 in volume in China (Shi et al., 2009, 2008). About 59% of them are distributed in Central Asia and Northwest China. They are typical Continental glaciers (also called “cold glaciers”). Glaciers there are a vital source of water for more than 130 million people and for wildlife ecosystems. Along with the global warming, glaciers there are experiencing severe retreat and shrinkage (Li et al., 2010). There are two relevant scientific topics triggering people’s interest most, one is the total volume of water stored in glaciers, and the other is the future contribu-

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*Corresponding author: lihuilin@lzb.ac.cn

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tion of melt water by glaciers.

A sound knowledge of the ice-thickness distribution on a glacier is essential for glacier volume calculation and future glacier change prediction. The total ice volume defines the amount of water stored by glaciers in a given catchment, and the ice-thickness distribution exerts an influence on the hydrological characteristics of the basin. Most studies on the impact of climate change on the hydrology of high alpine catchments (e.g., Huss et al., 2008), and most glacio-dynamical models (e.g., Aðalgeirsdóttir et al., 2006; Björnsson et al., 2006; Huybrechts and De Wolde, 1999; Hubbard et al., 1998; Oerlemans et al., 1998) require the ice-thickness distribution as an initial condition. However, direct measurement techniques of ice thickness, such as radio-echo sounding or borehole measurements, are expensive, laborious, and not feasible everywhere and necessarily restricted to a small number of glaciers. For studies focusing on large samples of glaciers, it is necessary to develop alternative approaches that are based on readily available datasets. At present, the total ice volume of glaciers is often estimated using volume-area scaling relations (Radic and Hock, 2010; Bahr et al., 1997; Chen and Ohmura, 1990; Driedger and Kennard, 1986), though the validation of the method is still under debate (Arendt et al., 2006). Several attempts that have been made to infer the ice thickness distribution by complex procedures, such as inverse methods based on modeling (Raymond and Gudmundsson, 2009; Thorsteinsson et al., 2003), which requires various inputs, and is not fit for studies on large samples of glaciers. The method, based on shallow-ice approximation and the assumption that ice behaves like perfect plasticity, is widely used to calculate thickness on ice-sheets (Mayer and Siegert, 2000; Paterson, 1994; Beget, 1987; Reeh, 1982). It was also used on mountain glaciers; however, the influence of valley drag was not taken into consideration or taken as constant (Paul and Svoboda, 2009; Oerlemans, 1997; Haeberli and Hoelzle, 1995; Huybrechts et al., 1989; Driedger and Kennard, 1986; Gerrard et al., 1952).

We present a method for estimating the ice-thickness distribution along flow-line of mountain glaciers, based on shallow-ice approximation and the assumption of perfect plasticity behavior of glacier ice. The distinction of the method presented here is that the drag of valley is fully considered, which promisingly gives more accurate results for mountain glaciers, especially for valley glaciers. For extension, a method to estimate the overall ice volume is also presented. Both of the methods to estimate thickness distribution and volume of glaciers are physically based and are quite easy to use; moreover, we do not need direct thickness measurements.

METHODOLOGY

Thickness along Flow-Line Calculation

Paterson (1970a, b), building upon the work of Nye (1952), suggested that it was possible to estimate glacier thickness by using glacier slope and an assumed constant shear for an infinitely wide glacier, with laminar flow. From the relation

$$h=\tau_b/(\rho g \sin\alpha) \quad (1)$$

where τ_b is the basal shear stress, ρ is the density of ice, g is the acceleration of gravity, h is ice thickness, and α is the surface slope. An assumption in the above relation is that the shear stress at the base τ_b equals to yield stress τ_y of ice.

Nye (1964) found numerical solutions for the steady rectilinear flow of ice, obeying Glen's nonlinear flow law, down uniform cylindrical channels of rectangular, semielliptical, and parabolic cross-section. According to his analysis, because the valley walls support part of the weight of the glacier, the basal shear stress on the centerline is less than its value for a very wide channel. Thus, a shape factor f is introduced to correct the calculation of basal shear stress τ_b as shown in the following equation

$$\tau_b=f\rho g h \sin\alpha \quad (2)$$

The value of f is dependent on the shape of valley cross-sections.

Glacial and/or glaciated trough cross-sections are generally referred to as being U-shaped or parabolic (Harbor and Wheeler, 1992; Harbor, 1990; Hirano and Aniya, 1988). It was Svensson (1959) who first introduced the power law equation as a mathematical function to represent the glacial trough cross-profile

$$y=ax^b \quad (3)$$

where x and y are the horizontal and vertical distances from the lowest point of the cross-section with a and b being constants. b is commonly used as an index of the steepness of the valley side, and a measure of the breadth of the valley floor. Since Svensson (1959), the power law equation has become a common tool in analyzing the morphology of glacial trough cross-profiles (Doornkamp and King, 1971). It has been found that b values range from less than 1 to over 5, with most values between 1.5 to 2.5, which is very close to parabolic form (b=2.0) (Li et al., 2001b; Aniya and Welch, 1981; Doornkamp and King, 1971; Graf, 1970). In addition, the validation of parabolic form (b=2.0) is proven by the variation principle, assuming that the glacier erosion works towards minimizing the friction between ice and bedrock (Hirano and Aniya, 1988). The values of f for glaciers flow in parabola profile valley are given in Table 1 (modified from Nye (1964), Table IV therein).

Table 1 The shape factor for parabolic profile valley (W=half-width/thickness on centerline)

W	f
0	0
1	0.445
2	0.646
3	0.746
4	0.806
∞	1.000

The values of f in Table 1 can be fit by equation (4)

$$f(W)=1-1/(1+mW) \tag{4}$$

where m=0.9 (R^2=0.999; P=0.01), and W is the ratio between half-width and thickness on the centerline. Nye (1964) also supplied values of f for elliptic and parabolic profiles. Based on comparison, the values are found only differ by about 9 percent on the average. In this case, values of f for parabolic profile are used to any valley section with power law function profiles. This is admittedly an approximation, but it is a valid step to establish a method to estimate ice distribution and volume of glaciers not requiring very high accuracy.

Another way to derive a shape factor is $f=A_c/(hP)$, where A_c is the cross-sectional area, and P is the perimeter length excluding the surface (i.e., the perimeter of constant between ice and bed) (Budd, 1969). The values of f obtained from two methods are the same for W=1, and only differ by about 10 percent on the average (Budd, 1969, page 45 therein). In this study, f in Table 1 is used in the following analysis.

Putting equation (4) into equation (2), the thickness along flow-line in a section can be expressed as

$$h_0=mwL/(mw-L) \tag{5}$$

where h_0 is the thickness along flow-line, w is the half width of a cross-section on surface, and $L=\tau_y/(\rho g\sin\alpha)$. The impact of basal sliding to the basal shear stress is not considered, and the estimation of uncertainties given rise by it is required by future studies.

Volume Calculation

The calculated ice thickness along flow-line is used as the main input for glacier volume calculation. Before calculating the volume, we try to eliminate one parameter from equation (3), to enable the profile function of sections to be determined by one set of measured data. Two models used in this study (Fig. 1), (1) assume that all cross-sections are parabolic

$$y=ax^2 \tag{6a}$$

and (2) use the obtained relationship between A ($\ln a=A$) and b ($|A|$=6.582b–6.133) by Li et al. (2001a)

$$y=\exp(6.582b-6.133)x^b \tag{6b}$$

The volume assessment can be divided into four steps as follows.

(1) Using the given glacier surface topography (slopes and widths), the ice thickness distribution along flow-line is calculated for each cross-section by equation (5). The interval of cross-sections between 10 and 50 m can be chosen, depending on the resolution of input data and glacier geometry.

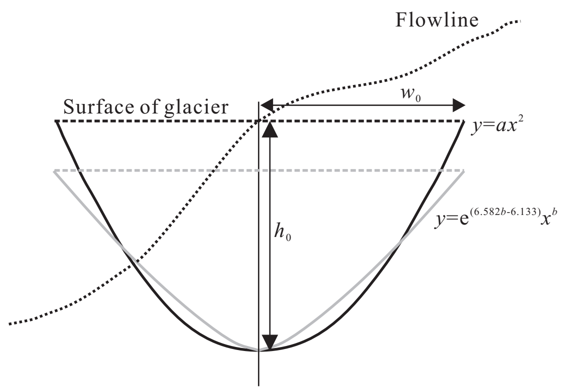

Figure 1. Different profiles of valley cross-section corresponding to equations (6a) and (6b).

(2) By introducing the calculated thickness on flow-line and measured half-width of a cross-section into equation (6a) or (6b), the coefficient *a* or *b* for each cross-section is identified, thus the profiles of cross-sections, the thickness distribution, as well as the area of each cross-section are obtained. The area of cross-section (A_{cs}) can be calculated in two ways according to equations (6a) and (6b). They are expressed as

$$A_{cs}=2aw^3/3 \tag{7a}$$

where $a=mL/(mw^2–wL)$.

$$A_{cs}=\exp(6.582b–6.133)w^{b+1}/(b+1) \tag{7b}$$

where $b=\{\ln[Lwm/(mw–L)]+6.133\}/[\ln(w)+6.582]$

(3) The area of cross-sections calculated in step (2) is used as input for an interpolation along flow-line. The interpolation routine uses an inverse distance averaging technique.

(4) The volume of glaciers is calculated by multiplying the mean area of each pair of neighboring cross-sections with corresponding intervals, *l* (defined as length of flow-line between two cross-sections).

Another way to calculate volume is like the following

(5) The same as step (1).

(6) The ice thicknesses at specific points at the surface of a cross-section can be calculated by equation (6a) or (6b), with the width and maximum thickness of the cross-section as inputs (Fig. 2).

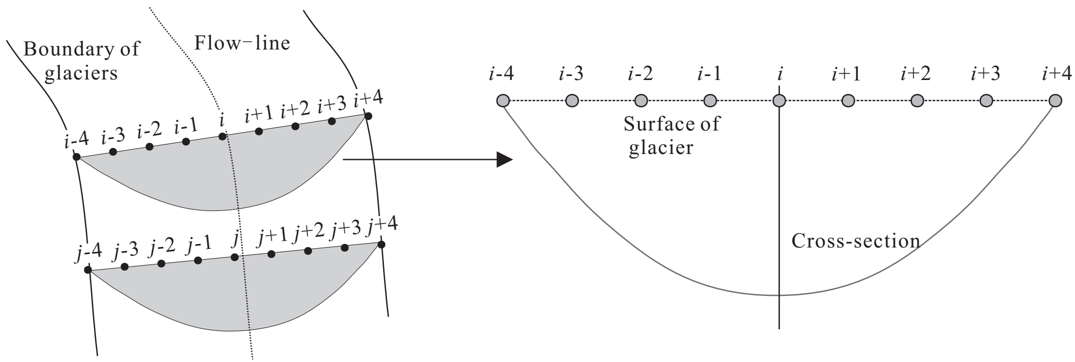

Figure 2. Specific points at the surface of a cross-section and their corresponding position on the surface of a glacier.

(7) Regular grids are needed to make on glacier surface. Grid spacing between 10 and 50 m can be chosen, depending on the resolution of input data and glacier size. The ice thickness calculated in step (2) is used as input for a spatial interpolation at each grid cell. The glacier outline is used as a boundary condition with zero ice thickness. The interpolation routine uses an inverse distance averaging technique, weighting the individual interpolation nodes with the inverse of the squared distance from the considered point. Finally, the calculated ice thickness is smoothened with a two-dimensional (2-D) discrete Gaussian filter of constant extension.

(8) The volume of glaciers is calculated by multiplying the area of each grid cell with corresponding ice thickness.

The second method is considered as better one than the first method. The reason of that will be presented in the following analysis.

DISCUSSION

Selection of Width of Cross-Sections

As noted in “Thickness along Flow-Line Calculation” section, the valley walls support part of the weight of the glacier, and the basal shear stress on the center-line is less than its value for a very wide channel. Thus, a shape factor *f* is introduced to correct the calculation of basal shear stress τ_b, and the half width of cross-sections is a significant input to estimating *f*. In this case, the value of width has impact on equations (5), (7a), and (7b). Usually, the width is determined by the intersection of the perpendicular line to flow-line and the glacier outline. This means the higher part of valley wall, with very thin ice lying on, is also involved in calculation by equations (5), (7a), and (7b); however, this part of valley wall contributes ignorable support to the weight of the glacier (Fig. 3a). In order to eliminate the corresponding error, the ice thickness calculated in step (1), and (1) is normalized with the local glacier width relevant for the ice discharge (here called 'effective width'). The effective width is determined along cross-sections perpendicular to the ice flow-lines and is based on the surrounding topography (Fig. 3a). The local glacier width is therefore reduced to the width for which the slope of the ice surface does not exceed a given threshold α_{lim}.

The profile of the cross-section of a glacier illustrated in Fig. 3 is representative for the valley glaciers. Because orientations of two sides of valley wall are opposite, the solar radiation and shading on ice lying on different side are different, which causes different ablation and accumulation. Thus, one side of cross-sections with much more ice accumulation than another side is a logical phenomenon according to above reasons. Furthermore, the 'biased' profile of cross-sections makes the flow-line deviate gradually from the middle longitudinal profile of the valley and the erosion of bed more serious at one side, which gives birth to an even more asymmetric profile of cross-sections.

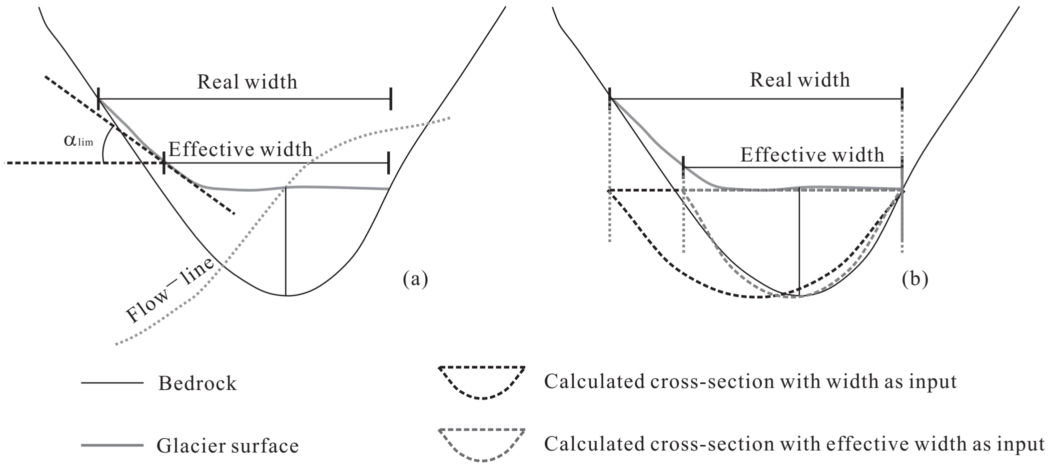

Figure 3. (a) Illustration of the difference between width and effective width of a cross-section, and how to determine effective width according to the surface geometry of a glacier; (b) illustration of the calculated profiles with different input.

In equations (5), (7a), and (7b), two hypotheses were premade: (1) the cross-section is symmetric in terms of flow-line; (2) the intersection of a cross-section and glacier surface (the surface line of a cross-section) is horizontal, which means that any point in the same surface line of a cross-section should have similar elevations. On real glaciers, both hypotheses do not always hold well, just like in Fig. 3. This causes that equations (5), (7a), and (7b) cannot calculate the maximum thickness and area of the real cross-sections of glacier but of cross-sections with relatively ideal profiles instead. For example, in Fig. 3b, the thickness and area of the profile bordered with black-dash lines, which is different from the real cross-section apparently, which would be calculated if the width is introduced as input. However, if the effective-width was introduced, the profile bordered with gray-dash lines is calculated instead, which is closer to the real cross-section. It is found that the effective width is much better than width to make the profile of the cross-section involved in calculation more symmetric in terms of flow-line, though some part of the area of the real cross-section is still missed.

Selection of Volume Calculation Method

In "Volume calculation" section, two volume calculation methods were presented. The first one is relatively simpler to use than the second one. However, when the geometry of the glacier is complex, the first method is probably invalid. Definite another parameter "effective interval", l_{e}, as

$$l_{\mathrm{e}}=A_{\mathrm{s}}/(w_i+w_{i+1}) \tag{8}$$

where A_{s} is the area of the glacier surface bordered by cross-sections i and i+1, and w_i and w_{i+1} are half widths of corresponding cross-sections. l and l_{e} are both illustrated in Fig. 4. If l=l_{e}, the first volume estimating method is valid, and vice versa. It is easily found that when neighboring cross-sections are not parallel to each other, and the profile of cross-sections are not symmetric in terms of flow-line; the volume of glacier between two cross-sections cannot be calculated by multiplying the mean of the area of two neighboring cross-sections with *l*. Although the effective-width can be introduced as input to reduce the asymmetry of the profile involved in calculation, the first method is recommended to calculate the volume of glaciers with simple geometry, especially those with valley walls almost parallel to each other. For those with complex geometry, the second method will work better.

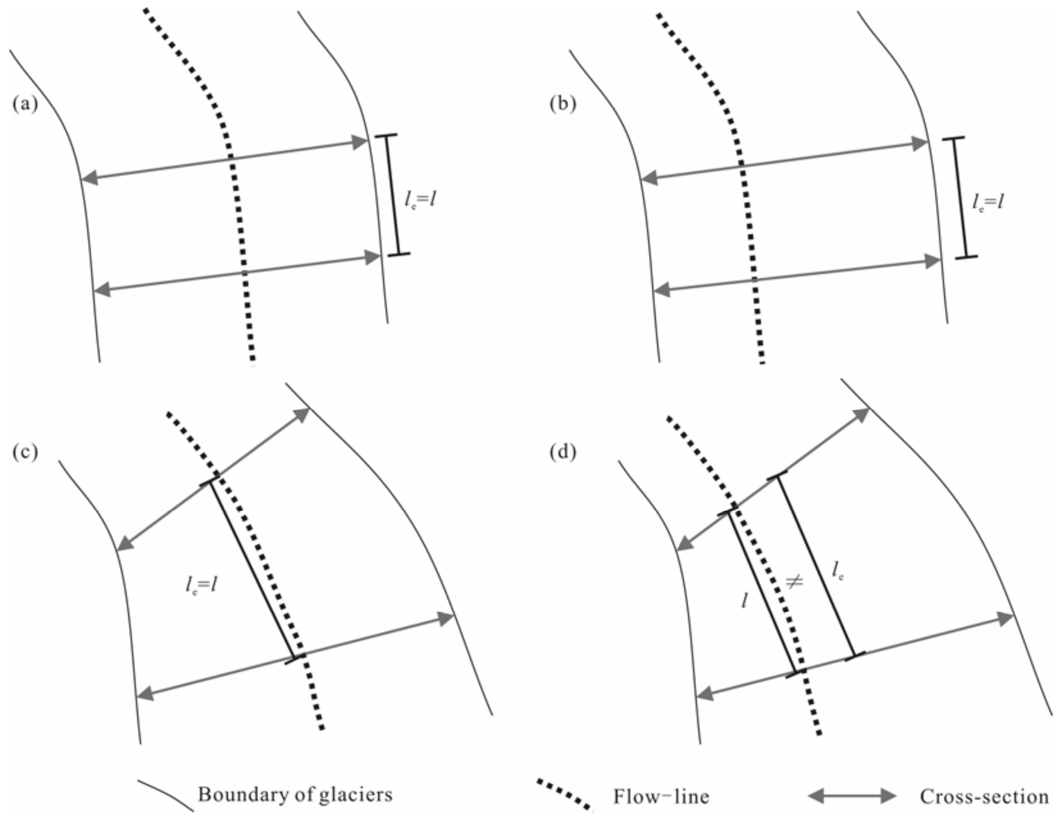

Figure 4. Illustration of real intervals and effective intervals between cross-sections according to different geometries of glaciers. (a) The neighboring cross-sections are parallel with each other, and the profiles of cross-sections are symmetric in terms of flow-line; (b) the neighboring cross-sections are parallel with each other, and the profiles of cross-sections are not symmetric in terms of flow-line; (c) the neighboring cross-sections are not parallel with each other, and the profiles of cross-sections are symmetric in terms of flow-line; (d) the neighboring cross-sections are not parallel with each other, and the profiles of cross-sections are not symmetric in terms of flow-line.

Limitation of Validation

Theoretically, it is impossible for Equation (5) to be valid for every glacier or every part of glaciers. The original form Equation (1) is investigated to search when and where Equation (5) is invalid. When the glacier ice behaves like perfect plasticity, and the ice flow is dominated by only one shear stress τ_{xy} (1-D SIA of full-stokes). Equation (1) can be used to calculate ice thickness (Paterson, 1994; Nye, 1964). It was successfully used on ice-sheets several times (Mayer and Siegert, 2000; Paterson, 1994; Beget, 1987; Reeh, 1982), giving evidence to that glacier ice behaves like perfect plasticity and to that the 1-D SIA is valid for a glacier with infinite width. For a valley glacier, the 1-D SIA can only work without evident error along flow-line (with a calibration factor *f*), and this is why Equation (5) in this study is restricted to thickness estimation along flow-line. When applied to ice sheets or glaciers, the theory boils down to the condition that the shear stress at the base of a glacier cannot exceed a certain yield stress (Oerlemans, 2008). The global glaciers retreat evidently in recent decades. Some valley glaciers, with very small scale and gentle slope of surface, are very thin at their tongues. The shear stress at the base of those glaciers is likely under the yield stress. Another condition to make Equation (1) invalid is that the surface of glacier is almost flat ($\alpha \approx 0°$), and then, the calculated thickness tends to be infinite. To solve the problem, a lower slope limit (e.g., α_0=5°) can be used for the surface slope filtering. However, the determination of the value of α_0 needs more discussion in future work.

Besides the theoretical hypotheses from Equation (1), another assumption introduced into Equation (5) is that the valley profile is parabolic. According to previous studies, the assumption holds well at most parts of valley glaciers. However, at the snout of some glaciers, the bed rock is flat, and the width can be regarded as infinite. In this case, Equation (1) is recommended to estimate thickness instead of Equation (5).

CONCLUSION

A method for estimating the ice-thickness distribution along flow-line of mountain glaciers, based on shallow-ice approximation and the assumption of perfect plasticity behavior of glacier ice, is built up. In the method, the impact of valley to glacier discharge is fully considered. Thus, the method is theoretically efficient for thickness estimation of mountain glaciers, especially for that of valley glaciers. Based on calculated thickness, a method to estimate the overall ice volume is established. Both the methods are physically based and simple to use. The input data needed are glacier geometry parameters: slope, width, length, and outline; all of which can be abstracted from topographic map or remote sensing image.

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